Math 10A with Professor Stankova
Quiz 12; Wednesday, 11/15/2017
Section #107; Time: 11 AM
GSI name: Roy Zhao **Name:** _____

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True **FALSE** If the mean of a distribution exists, then the standard deviation exists.

Solution: The mean can exist but sometimes the standard deviation doesn't.

2. True **FALSE** Chebyshev's inequality can help us when $0 < k < 1$.

Solution: When $k < 1$, we have that $1/k^2 > 1$ and hence $1 - 1/k^2 < 0$, so it doesn't help us.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (4 points) Calculate the standard deviation of $f(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & otherwise \end{cases}$.

The mean is $\frac{2}{3}$.

Solution: First we need to calculate the mean. The mean is

$$\int_{-\infty}^{\infty} xf(x)dx = \int_0^1 x(2x)dx = \frac{2x^3}{3}|_0^1 = \frac{2}{3}.$$

Then the variance is

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)dx - \frac{2^2}{3^2} = \int_0^1 2x^3 dx - \frac{4}{9}$$

$$= \frac{x^4}{2}|_0^1 - \frac{4}{9} = \frac{1}{2} - \frac{4}{9} = \frac{1}{18}.$$

So the standard deviation is $\frac{1}{\sqrt{18}}$.

(b) (4 points) Calculate the standard deviation of $\{0, 3, 3\}$.

Solution: The mean is $\frac{0+3+3}{3} = 2$. The variance is

$$\frac{(0-2)^2 + (3-2)^2 + (3-2)^2}{3} = \frac{4+1+1}{3} = 2.$$

So the standard deviation is $\sqrt{2}$.

(c) (2 points) Let f be a PDF with mean 0 and standard deviation 1. For what value of a can we say that $P(-a \leq X \leq a) \geq 0.99 = \frac{99}{100}$?

Solution: We know that $P(-a \leq X \leq a) = P(\mu - a\sigma \leq X \leq \mu + a\sigma) \geq 1 - \frac{1}{a^2}$. So we need that $0.99 = 1 - \frac{1}{a^2}$ so $a^2 = 100$ and $a = 10$.